Enhanced SHM Using Tensor Decompositions and Sparse Atomistic Models for Damage Monitoring of Aeronautical Composite Structures

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ABSTRACT

Developing robust Structural Health Monitoring (SHM) solutions for large structures, particularly in the aerospace sector, remains challenging due to the volume, variability, and complexity of the data involved. One very promising solution is based on active ultrasonic guided waves (UGW) which are signals emitted and received by a set of transducers bonded to the structure to monitor. However, existing SHM algorithms cannot solve the aforementioned challenges under the current paradigm of path-by-path processing of the raw UGW signals. To move forward, a new paradigm is introduced in this work. This new approach exploits the intrinsic multi-dimensional tensorial nature of SHM UGW data through Canonical Polyadic Decomposition (CPD) and couples it with the Single Atom Convolutional Matching Pursuit Method (SACMPM). This redefines classical sparse decomposition techniques building accurate and efficient wave propagation models tailored to SHM applications. A unique UGW database where regular ground-based measurements have been carried out on an actual A380 running flight test is described in order to challenge the proposed paradigm shift. The efficiency of the coupling between SACMPM and CPD is illustrated here with respect to their ability to compress UGW information, and extract meaningful information from UGW signals in a physically informed manner. Additionally, a CPD-based damage localization algorithm is enriched using a SACMPM decomposition. Extracted physically informed features can thus be efficiently used for physically informed data driven damage monitoring approaches. The proposed paradigm shift thus demonstrates a strong potential for scalable, transferable, and reliable UGW SHM solutions, bridging the gap between laboratory experiments and real-world deployment. This paradigm shift is also expected to inspire further research and innovative ideas, leading to breakthroughs in the adoption of active UGW signals for SHM applications.

KEYWORDS: *Ultrasonic Guided Waves, Canonical Polyadic Decomposition, Single Atom Convolutional Matching Pursuit, Data Compression*

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INTRODUCTION

Developing robust Structural Health Monitoring (SHM) solutions for large structures, particularly in the aerospace sector, is increasingly challenged by the volume, variability, and complexity of the data. While extensive experimental data are available for healthy structural states, data for damaged states remain limited and often require computationally intensive simulations. In addition, environmental and operational variations - such as temperature and loading - add further complexity to signal interpretation and model reliability. Ultrasonic guided waves (UGW) emitted and received by a set of transducers bonded to the structure to monitor appears to be a promising means of solving those issues. Numerous algorithms for detecting, locating, classifying and quantifying damage in composite structures using UGW have already been proposed in the literature [1]. Within the current paradigm of processing the raw UGW signals acquired path-by-path, the proposed SHM algorithms are not satisfying at resolving the problems.

Despite several decades of research on SHM systems, the achievements are still largely academic, due to difficulties transporting results from the laboratory to real systems. To move one step further out of the laboratory, a paradigm shift is introduced here. The core idea is a hybrid methodology integrating physical knowledge, machine learning and advanced signal representations to improve the performance of UGW-based SHM under realistic conditions. At first, we introduce the Single Atom Convolutional Matching Pursuit framework, which redefines classical sparse decomposition techniques to construct accurate and efficient wave propagation models that are tailored to SHM applications involving the emission of a single tone burst in structures for monitoring purposes [1]. Secondly, we exploit the intrinsic multi-dimensional tensorial nature of SHM UGW data through canonical polyadic decomposition. This enables compact, interpretable representations that improve damage monitoring by naturally considering data from all available UGW propagating paths [2]. Additionally, we describe a unique UGW data base where regular ground measurements have been achieved on an actual A380 running flight tests [3] in order to evaluate the proposed paradigm shift's ability to compress UGW information in a physically informed manner. Finally, a CPD-based damage localization algorithm [2] is enriched using a SACMPM decomposition to illustrate potential applications of the proposed paradigm.

We therefore expect the proposed paradigm shift to demonstrate significant potential for scalable, transferable, and reliable SHM, bridging the gap between laboratory scale experiments and real-world deployment. We also expect the paradigm shift to inspire further research works and to stimulate innovative ideas leading to breakthroughs in the use of UWG signals for SHM applications.

ATOMISTIC GUIDED WAVES SIGNALS DECOMPOSITION

UGWs are bending and compression waves that stress the entire thickness of the monitored thin structure. These waves can propagate over relatively large distances and therefore cover a large control surface with few transducers in a short time [4]. However, UGW has two main drawbacks: at any given frequency, at least two modes coexist simultaneously, and these modes are dispersive. This means that UGW velocities depends on the frequency, which makes interpreting the collected signals tricky in practice, despite being extremely informative regarding the monitored structure health.

The basic idea underlying UGW-based SHM is to use a tone burst signal centered around a given frequency to excite a transducer bonded to the structure being monitored. This Initial Wave Packet (IWP) then propagates through the inspected structure and interacts with its boundaries, structural discontinuities, and eventual damage. Each of these discontinuities produces an additional wave packet propagating within the host structure. The resulting signals measured by the other transducers correspond to the IWP after propagation within the host structure and multiple interactions caused by structural discontinuities. UGW-based SHM algorithms seek to detect echoes caused by the presence of damage in these signals in order to infer damage presence, location, type, and severity. In such a context, the ability to decompose the measured signals into wave packets that can be physically interpreted and potentially linked to structural damage is thus of great interest.

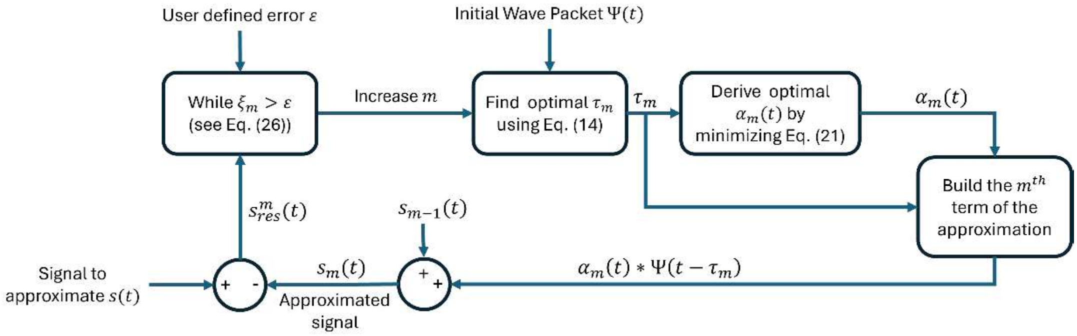

Figure 1: Overview of the SACMPM method for the decomposition of UGW signals [1]. The signal to estimate $s(t)$ is iteratively approximated by the convolution of an optimal kernel $\alpha_m(t)$ with a delayed version of the IWP $\Psi(t - \tau_m)$ until the residue is lower than a given user defined error or the maximum number of terms is reached.

To take advantage of this fact, an improved version of matching pursuit was proposed [1]. It addresses the decomposition of UGW-based SHM signals by decomposing a measured signal into delayed and dispersed impulse responses derived from a single atom. This decomposition is called the Single Atom Convolutional Matching Pursuit Method (SACMPM) and is illustrated in Figure 1. The decomposition is achieved through a purely mathematical construction, without the need for an over-represented learning dictionary. Only a single atom $\Psi\ (t)$ is used, representing the external excitation imposed on the structure and corresponding to the Initial Wave Packet (IWP). Such a decomposition can be obtained numerically by following a greedy process that builds the decomposition on-the-fly until convergence. Dispersion effects can be introduced through a convolution operation. This method has been applied to experimental UGW-based SHM signals for comparison purposes and highlighted the benefits it offers. Damage localization has also been achieved using machine learning algorithms fed by features extracted from such decomposition thus demonstrating its practical interest for SHM purposes [1].

TENSORIAL APPROACHES FOR GUIDED WAVES SIGNALS

Considering a smart structure to be monitored equipped with N transducers and for which acquisition is performed over K samples, one naturally ends up with a matrix $\boldsymbol{M} \in \mathbb{R}^{N\times N\times K}$ at the end of a pitch-catch SHM process (with the "*actuator*", "*sensor*", and "time" dimensions). The potential occurrence of damage is monitored by first taking measurements in a reference state to create a reference matrix $\boldsymbol{R}$. Then, as the structure's life cycle progresses, measurements are taken at various unknown states, resulting in the matrix, $\boldsymbol{U}$. The matrix $\boldsymbol{\delta}$ that corresponds to the difference between $\boldsymbol{R}$ and $\boldsymbol{U}$ is the basis of the detection, localization, classification, and quantification steps of SHM. The resulting matrices $\boldsymbol{U}, \boldsymbol{R}, \boldsymbol{\delta} \in \mathbb{R}^{N\times N\times K}$ lie along three dimensions and can thus be interpreted as three-way tensors [5, 6, 7]. SHM data are therefore highly redundant and correlated, and path-by-path approaches promoted in the literature cannot handle all these relationships. Tensors then emerge as an alternative tool able to manage SHM data all at once. Adequate and unified data analysis (in opposition to previously mentioned "*path-by-path*" analysis) could then be carried out to highlight the underlying structure of SHM data and thus potentially perform damage monitoring. The aim is to identify interesting structures and features popping out of the tensors associated with UGW SHM data in a physically informed manner.

From tensors literature, it is well known that tensors can be decomposed using the Canonical Polyadic Decomposition (CPD) up to a rank R [5, 6, 7]. For a three-way tensor, such decomposition consists in finding a triplet ($\boldsymbol{a} \in \mathbb{R}^{N\times R}, \boldsymbol{b} \in \mathbb{R}^{N\times R}, \boldsymbol{c} \in \mathbb{R}^{K\times R}$) that allows for a more compact representation of a given tensor (see Figure 2).

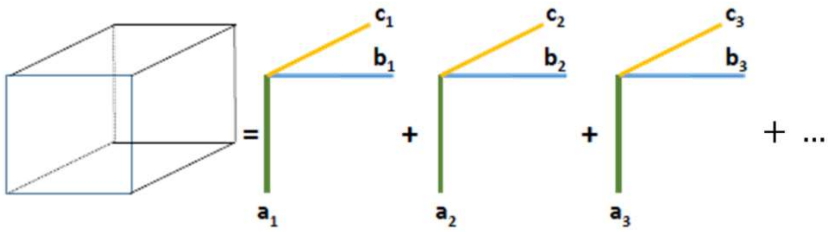

Figure 2: Schematic representation of the CPD of 3D-tensors

Following the notations of Figure 2, the CPD of an UGW tensor $\boldsymbol{U}$ can be expressed as:

$$U_{nmk} = \sum_{r=1}^{R} a_{rn}^{\Phi} b_{rm}^{\Phi} c_{rk}^{\Phi} \Phi_{nmk}$$

where U_{nmk} is the coefficient corresponding to indexes n, m, and k of the 3D tensor $\boldsymbol{U}$.

AN UNIQUE A380 DATABASE

A fan cowl structure (FCS) mounted on an instrumented A380 plane is employed to illustrate the proposed paradigm. This structure is made up of a four-layered carbon epoxy composite plate with stacking sequence [0°/-45°/+45°/0°]. The transducers deployments is shown in Figure 3. There is in total 43 transducers installed on the mounted FCS, respectively.

Figure 3: PZT deployments on the Fan Cowl Structure on an instrumented A380.

During testing, a signal generator produced a five-cycle sinusoid tone burst signal modulated by Hanning window given that this kind of exciting signal is a standard in SHM of composite structures. The central frequency of the excitation signal was $100\,kHz$ and the sampling frequency was set as $1\,MHz$. Among the transducers, each one was used as an actuator in a round robin fashion and the remaining others were receivers, i.e. a sequential pitch-catch testing scheme was conducted. Representative UGW signals acquired during the experimental campaign for PZT 16 to 20 are shown in Figure 4. The diagonal line shows the emission signal also being recorded and used as the initial wave packet (IWP) in the SACMPM decomposition [1].

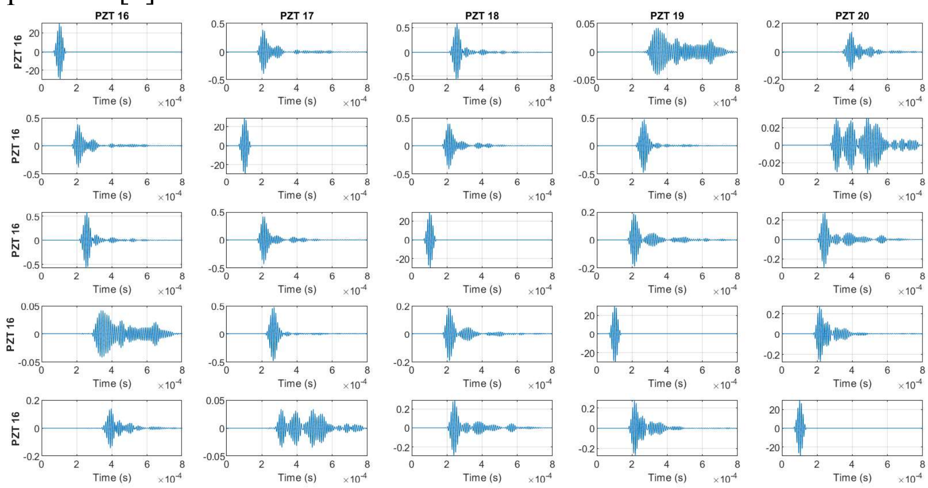

Figure 4: Representative UGW acquired on ground on the A380 Fan Cowl Structure.

UGW DATA COMPRESSION USING SACMPM AND CPD

The idea is now to demonstrate that the proposed approach considering the fact that all the collected signals stem from one unique wave packet (SACMPM) is efficient for SHM purposes. This approach also considers that all signals correspond to a unique transducer network leading to a tensorial representation (CPD). To illustrate this, we demonstrate in the sequel, that efficient physically informed UGW data compression can be achieved on the previously introduced dataset thanks to SACMPM and CPD.

EXPERIMENTAL SIGNALS DECOMPOSITION USING SACMPM

Atomic decomposition is first applied to the experimental signals using either convolution of the IWP (SACMPM) or using only delayed versions of the IWP (SAMPM). Those two approaches have been tested on the experimental signals collected on the FCS of the A380 being monitored.

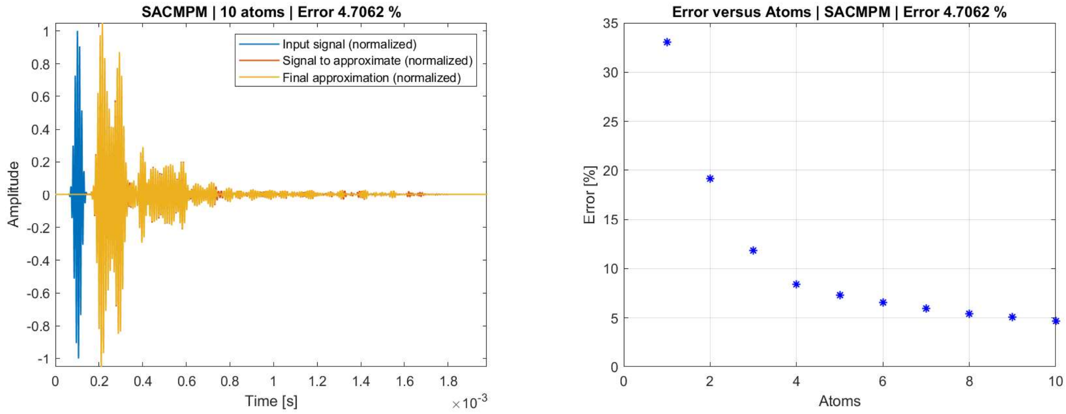

Figure 5: Application of SAMPM to the signal corresponding to the path going from actuator 16 to sensor 17.

The SACMPM decomposition algorithm was applied on the signal corresponding to the path going from actuator 16 to sensor 17 and the obtained results are shown in Figure 5. Using a set of 35 delayed and scaled versions of the input signal enables the experimentally measured signal to be approximated with an error less than 5% in energy. The visual comparison of the original and approximated signals shows that this error rate is very low in practice. The data compression ratio is very high here as only the input signal and the 35 associated delays and convolutions kernels need to be stored. The SAMPM can also be applied on the same signal providing similar results but with an error or 10%.

APPLICATION OF CPD TO THE SIGNALS DECOMPOSITION

After applying the SAMPM or SACMPM to the signals, one obtains for N actuators, N sensors, and L atoms, a matrix $\boldsymbol{T} \in \mathbb{R}^{N\times N\times L}$ and a matrix $\boldsymbol{A} \in \mathbb{R}^{N\times N\times L}$ or $\boldsymbol{K} \in \mathbb{R}^{N\times N\times L\times M}$ (here $M = 40$ has been chosen) corresponding respectively to the delays and the amplitudes for SAMPM or convolutional kernels for SACMPM of the atoms. These matrices can be considered as 3-ways or 4-ways tensors and thus CPD can be applied to them up to a rank R as a second compression step. A $\boldsymbol{T}$ is encoding extremely fine and sensitive time related information, CPD has been applied only to $\boldsymbol{A}$ or $\boldsymbol{K}$ in the following. The implementation of CPD proposed in [8] is here being used.

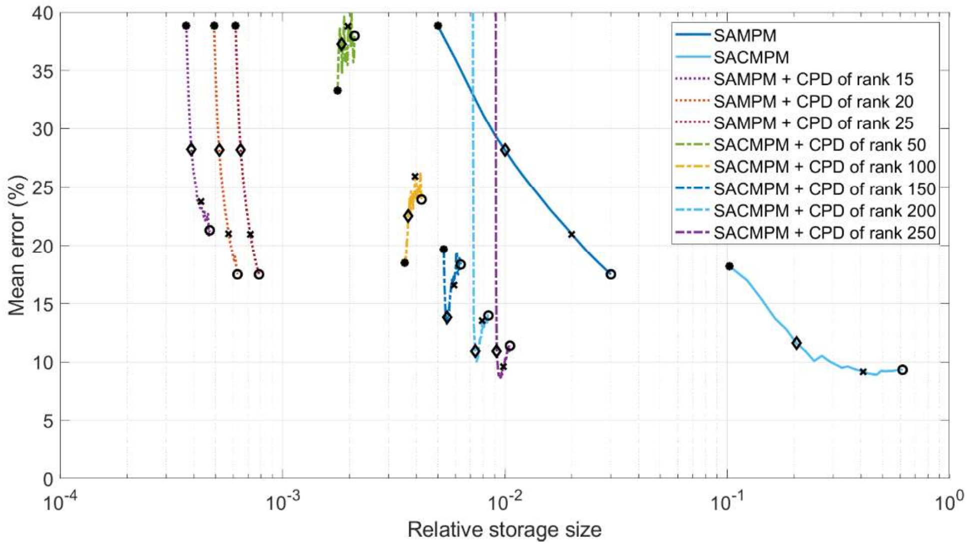

*Figure 6: Mean approximation error as a function of the relative storage size for SAMPM, SACMPM, SAMPM coupled with CPD, and SACMPM coupled with CPD. Plain circles for 5 atoms, diamonds for 10 atoms, "**x**" for 20 atoms and "**o**" for 30 atoms retained.*

For SAMPM, SACMPM, SAMPM with a CPD applied on $\boldsymbol{A}$, and SACMPM with a CPD applied on $\boldsymbol{K}$ the mean approximation error among pitch catches UGW signals for transducers from 16 to 20 have been computed. For SAMPM and SACMPM, the only parameter is the number of atoms retained. When CPD is applied, an additional parameter, the CPD rank, appears. Additionally, the relative storage size has been computed. Here, this is interpreted as the number of values to be stored after data reduction, compared with the initial number of values to be stored when using raw UGW signals. The relative storage size has been computed by extrapolating to consider the 43 transducers.

Figure 6 shows the evolution of error versus the relative storage size for the various methods being investigated. Initially, SAMPM appears to be much more effective than SACMPM in terms of compression abilities albeit at the expense of precision. Furthermore, whatever the retained method (except SACMPM coupled with CPD), the error lowers as the number of retained atom increases. Finally, the addition of CPD to SAMPM or SACMPM leads to a significant reduction in storage size without loss of precision. The proposed paradigm thus appears as relevant in the present context and deserves being exploited in further research work.

DAMAGE LOCALIZATION USING SAMPM AND CPD

The fact that Lamb wave SHM based data are naturally three-way tensors has previously been investigated by the authors for damage localization purpose [2]. Under classical assumptions regarding wave propagation, it was demonstrated that the canonical polyadic decomposition of rank 2 of the tensors built from the phase and amplitude of the difference signals between a healthy and damaged states that provides direct access to the distances between the transducers and damage. This property has been successfully used to propose an original tensor-based damage localization algorithm. Compared with standard damage localization algorithms (delay-and-sum, RAPID, and ellipse- or hyperbola-based algorithms) the proposed algorithm appears to be more precise and robust on the investigated cases. Furthermore, it is important to note that this algorithm only requires raw signals as inputs with no need for specific pre-processing steps or finely tuned external parameters.

A core step in this CPD-based algorithm is to be able to extract from the difference signals (i.e. the signals corresponding to the difference between the healthy and damage states) the time of arrival and the amplitude of the first wave packet generated by the damage to feed it to the tensor decomposition algorithm. This task is performed using a local maximum detection and a windowing around it and is consequently not extremely robust. SACMPM is naturally seeking to decompose signals in waves packets and is thus well suited to achieve that task. The tensorial-based damage localization method presented in [2] has thus been here enriched with SACMPM.

The tested case corresponds to an unmounted FCS structure equivalent to the one of Figure 3 and available at PIMM laboratory. UGW data have been collected for the 30 available PZT transducers for a healthy case and a case where a 6 mm hole has been drilled into the FCS. The CPD-based and SACMPM enriched damage localization algorithm derived from [2] has been applied to the collected data. Illustrative results are provided in Figure 7 demonstrating the validity of the proposed approach.

DISCUSSION

The coupling between the atomistic representation of UGW signals, as provided by SACMPM, and the tensorial view of the transducer network is firmly anchored in physics, making it highly relevant for algorithm design and UGW data processing. This paradigm shift has been demonstrated in terms of its ability to compress UGW data by a factor of 100 while maintaining accuracy, and in its provision of robust damage localisation algorithms. The features extracted in this way can be used more widely as physically informed features for machine learning, or to extend existing physically based algorithms by providing robust wave packet decomposition and a tensorial viewpoint.

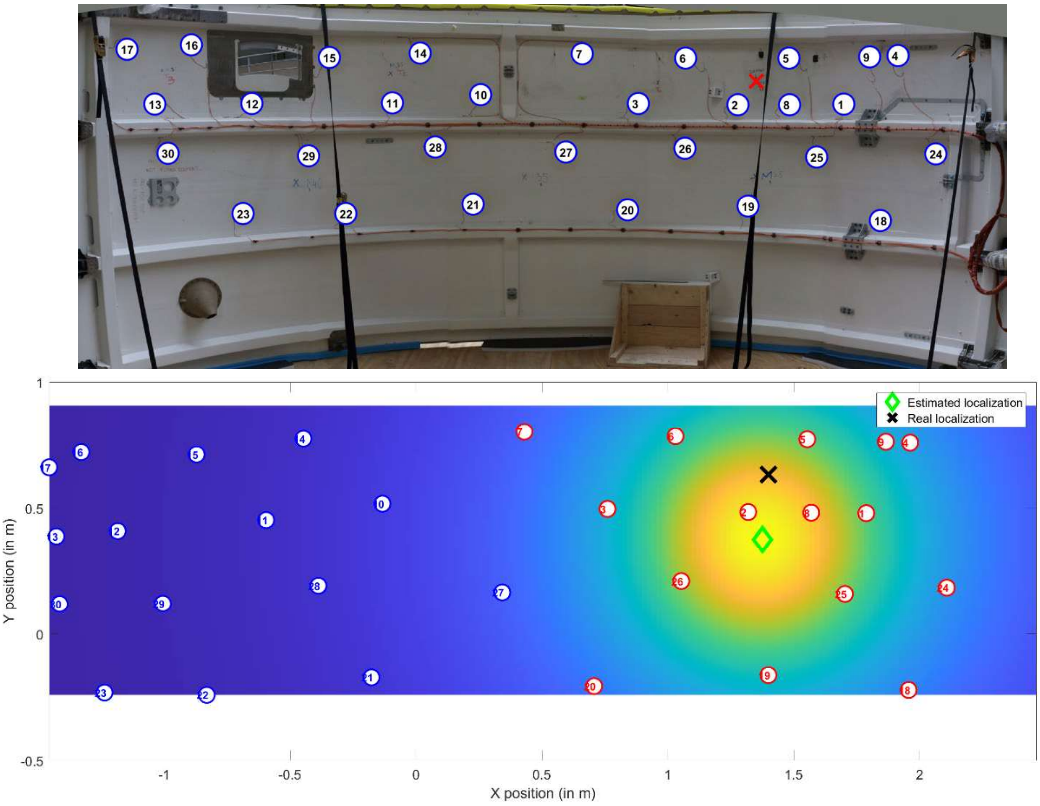

Figure 7: Illustration of the application of the tensorial-based damage localization method presented in [2] enriched with SACMPM. Top: FCS structure under inspection. Bottom: Damage localization results (UGW raw signal from transducers in red are used).

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